

Algebra I Introduction

The fundamental purpose of the Model Algebra I course is to formalize and extend the mathematics that students learned in the middle grades. This course is comprised of standards selected from the high school **conceptual categories**, which were written to encompass the scope of content and skills to be addressed throughout grades 9–12 rather than through any single course. Therefore, the complete standard is presented in the model course, with clarifying footnotes as needed to limit the scope of the standard and indicate what is appropriate for study in this particular course. For example, the scope of Model Algebra I is limited to linear, quadratic, and exponential expressions and functions as well as some work with absolute value, step, and functions that are piecewise-defined. Therefore, although a standard may include references to logarithms or trigonometry, those functions are not to be included in coursework for Model Algebra I; they will be addressed later in Model Algebra II. Reminders of this limitation are included as footnotes where appropriate in the Model Algebra I standards.

For the high school Model Algebra I course, instructional time should focus on four critical areas: (1) deepen and extend understanding of linear and exponential relationships; (2) contrast linear and exponential relationships with each other and engage in methods for analyzing, solving, and using quadratic functions; (3) extend the laws of exponents to square and cube roots; and (4) apply linear models to data that exhibit a linear trend.

- (1) By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. In Algebra I, students analyze and explain the process of solving an equation and justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating among various forms of linear equations and inequalities, and use them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations.
- (2) In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. In Algebra I, students learn function notation and develop the concepts of domain and range. They focus on linear, quadratic, and exponential functions, including sequences, and also explore absolute value, step, and piecewise-defined functions; they interpret functions given graphically, numerically, symbolically, and verbally; translate between representations; and understand the limitations of various representations. Students build on and extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.
- (3) Students extend the laws of exponents to rational exponents involving square and cube roots and apply this new understanding of number; they strengthen their ability to see structure in and create quadratic and exponential expressions. They create and solve equations, inequalities, and systems of equations involving quadratic expressions. Students become facile with algebraic manipulation, including rearranging and collecting terms, and factoring, identifying, and canceling common factors in rational expressions. Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.
- (4) Building upon their prior experiences with data, students explore a more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

Algebra I Overview

Number and Quantity

The Real Number System

- Extend the properties of exponents to rational exponents.
- Use properties of rational and irrational numbers.

Quantities

- Reason quantitatively and use units to solve problems.

Algebra

Seeing Structure in Expressions

- Interpret the structure of expressions.
- Write expressions in equivalent forms to solve problems.

Arithmetic with Polynomials and Rational Expressions

- Perform arithmetic operations on polynomials.

Creating Equations

- Create equations that describe numbers or relationships.

Reasoning with Equations and Inequalities

- Understand solving equations as a process of reasoning and explain the reasoning.
- Solve equations and inequalities in one variable.
- Solve systems of equations.
- Represent and solve equations and inequalities graphically.

Functions

Interpreting Functions

- Understand the concept of a function and use function notation.
- Interpret functions that arise in applications in terms of the context.
- Analyze functions using different representations.

Building Functions

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

Linear, Quadratic, and Exponential Models

- Construct and compare linear, quadratic, and exponential models and solve problems.
- Interpret expressions for functions in terms of the situation they model.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Statistics and Probability

Interpreting Categorical and Quantitative Data

- Summarize, represent, and interpret data on a single count or measurement variable.
- Summarize, represent, and interpret data on two categorical and quantitative variables.
- Interpret linear models.

★ Indicates a modeling standard linking mathematics to everyday life, work, and decision-making
(+) Indicates additional mathematics to prepare students for advanced courses.

Algebra I

Number and Quantity

The Real Number System

N-RN

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Quantities

N-Q

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★
2. Define appropriate quantities for the purpose of descriptive modeling. ★
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. ★

Algebra

Seeing Structure in Expressions

A-SSE

Interpret the structure of expressions [In Algebra I these standards address linear, exponential, and quadratic expressions.]

1. Interpret expressions that represent a quantity in terms of its context. ★
 - a. Interpret parts of an expression, such as terms, factors, and coefficients. ★
 - b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P .* ★
2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*
 - ~~a. Use the distributive property to express a sum of terms with a common factor as a multiple of a sum of terms with no common factor. For example, express $xy^2 + x^2y$ as $xy(y + x)$. (Common Core Standard A-SSE-2a)~~
 - ~~b. Use the properties of operations to express a product of a sum of terms as a sum of products. For example, use the properties of operations to express $(x + 5)(3 - x + c)$ as $-x^2 + cx - 2x + 5c + 15$. (Common Core Standard A-SSE-2b)~~

~~2.1 Apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials. (CA Standard Algebra I - 11.0)~~

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★
 - a. Factor a quadratic expression to reveal the zeros of the function it defines. ★
 - b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. ★

- c. Use the properties of exponents to transform expressions for exponential functions. *For example, the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. ★*

Arithmetic with Polynomials and Rational Expressions

A-APR

Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials **(revised)**.

Creating Equations

A-CED

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable **including ones with absolute value** and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. **(revised)** ★ [In Algebra I this standard addresses linear, exponential, and quadratic equations and inequalities]
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★ [In Algebra I this standard addresses graphing with only 2 variables] [In Algebra I this standard addresses linear, exponential, and quadratic functions]
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* ★ [In Algebra I this standard addresses linear and quadratic systems of equations.]
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R .* ★ [In Algebra I this standard addresses linear, exponential, and quadratic formulas.]

Reasoning with Equations and Inequalities

A-REI

Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable.

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

3.1 Solve one-variable equations and inequalities involving absolute value, graphing the solutions and interpreting them in context. (revised) [In Algebra I this standard addresses linear functions.]

4. Solve quadratic equations in one variable.
 - a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
 - b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .

Solve systems of equations.

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

- Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*

Represent and solve equations and inequalities graphically. [In Algebra I, these standards address linear and exponential systems]

- Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★
- Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. [In Algebra I, this standard addresses linear inequalities]

Functions

Interpreting Functions

F-IF

Understand the concept of a function and use function notation.

- Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
- Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for $n \geq 1$.*

Interpret functions that arise in applications in terms of the context.

- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★ [In Algebra I this standard addresses linear, exponential, absolute value and quadratic functions.]
- Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ★ [In Algebra I this standard addresses linear, exponential, absolute value and quadratic functions.]
- Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

Analyze functions using different representations.

- Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★
 - Graph linear and quadratic functions and show intercepts, maxima, and minima. ★
 - Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ★ [In Algebra I step functions are not addressed]

- e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. ★ [In Algebra I, this standard addresses only exponential functions]
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
 - a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
 - b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, and $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.*
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* [In Algebra I this standard addresses linear, exponential, quadratic, absolute value and piecewise defined functions.]

Building Functions

F-BF

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities. ★
 - a. Determine an explicit expression, a recursive process, or steps for calculation from a context. ★
 - b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* ★
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ★

Build new functions from existing functions.

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* [In Algebra I this standard addresses linear, exponential, quadratic, and absolute value functions.]
4. Find inverse functions.
 - a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ or $f(x) = (x + 1)/(x - 1)$ for $x \neq 1$.* [In Algebra I, this standard addresses linear inverses only]

Linear, Quadratic, and Exponential Models

F-LE

Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions. ★
 - a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. ★
 - b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. ★
 - c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. ★
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). ★
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. ★

Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context. ★
6. **Apply quadratic functions to physical problems, such as the motion of an object under the force of gravity.** ★ (revised)

Statistics and Probability

Interpreting Categorical and Quantitative Data

S-ID

Summarize, represent, and interpret data on a single count or measurement variable.

1. Represent data with plots on the real number line (dot plots, histograms, and box plots). ★
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. ★
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★

Summarize, represent, and interpret data on two categorical and quantitative variables.

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. ★
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. ★
 - a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* ★
 - b. Informally assess the fit of a function by plotting and analyzing residuals. ★
 - c. Fit a linear function for a scatter plot that suggests a linear association. ★

Interpret linear models.

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ★
8. Compute (using technology) and interpret the correlation coefficient of a linear fit. ★
9. Distinguish between correlation and causation. ★