Orange Unified School District

ADVANCED ALGEBRA WITH FINANCIAL APPLICATIONS

Year Course

GRADE LEVEL: 11-12

PREREQUISITES: Algebra I and Geometry with "C" grade or better

INTRODUCTION TO THE SUBJECT:

Advanced Algebra with Financial Applications is a mathematical modeling course that is algebra-based, applications-oriented, and technology-dependent. The course addresses college preparatory mathematics topics from Advanced Algebra, Statistics, Probability, Precalculus, and Calculus under seven financial umbrellas: Banking, Investing, Credit, Employment and Income Taxes, Automobile Ownership, Independent Living, and Retirement Planning and Household Budgeting. The course allows students to experience the interrelatedness of mathematical topics, find patterns, make conjectures, and extrapolate from known situations to unknown situations. The mathematics topics contained in this course are introduced, developed, and applied in an as-needed format in the financial settings covered. Students are encouraged to use a variety of problem-solving skills and strategies in real-world contexts, and to question outcomes using mathematical analysis and data to support their findings. The course offers students multiple opportunities to use, construct, question, model, and interpret financial situations through symbolic algebraic representations, graphical representations, geometric representations, and verbal representations. It provides students a motivating, young-adult centered financial context for understanding and applying the mathematics they are guaranteed to use in the future, and is thusly aligned with the recommendations of the Common Core State Standards, as stated in this excerpt:

"...all students should be strongly encouraged to take math in all years of high school. ...An array of challenging options will keep math relevant for students, and give them a new set of tools for their futures…" From the Common Core State Standards

Advanced Algebra with Financial Applications offers 11th and 12th grade students an opportunity to view the world of finance through a mathematical lens. The topics were developed using the Common Core State Standards in Mathematics, the California Mathematics Standards, and the NCTM Curriculum and Evaluation Standards. The mathematical formulas, functions, and pictorial representations used assist students in making sense of the financial world around them and equip them with the ability to make sound financial decisions.

The overarching purpose of the course is to develop the type of mathematically proficient students addressed in this excerpt from the Common Core State Standards for Mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. Mathematically proficient students who can *apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.*

Advanced Algebra with Financial Applications builds strength in reasoning and number sense, because the real-world applications demand that solutions make sense. Through contextual problem solving and the mathematical modeling of real situations, the course gives the students the motivation to persevere through routine and non-routine problems, and as a result, develop strength and confidence in their mathematics ability.

COURSE OUTLINE:

Unit 1: Banking Services

In this unit, students use exponential functions to compute compound interest and compare it to simple interest. They derive formulas and use iteration to compute compound interest. They apply their findings to short-term, long-term, single deposit and periodic deposit accounts.

Mathematics Topics

- Derivation of the compound interest formula
- Exponential functions
- Computations based on iterative processes
- Limits of polynomial functions, rational functions, and sequences
- Natural logarithm as the inverse of the exponential function
- Exponential growth and decay
- Solving exponential equations
- Using inductive reasoning

Mathematics Learning Goals

- Students will use the simple interest formula $I = PRT$ and using inverse operations to solve for all four variables.
- Students will use iteration to show how compounding pays "interest on your interest."
- Students will derive the compound interest formula $B = (1+\frac{r}{n})^{nt}$ by using patterns and inductive reasoning.
- Students will compute compound interest with and without the formula.
- Students will apply and interpret the limit notation $\lim_{x\to a} f(x) = b$.

- Students will model an infinite series and finding a finite sum for an infinite series with common ratio ½.
- Students will compute limits of polynomial functions as $x \to \infty$.
- Students will approximate the natural base ***e*** by examining the sequence $\left\{\left(1+\frac{1}{x}\right)^x\right\}$ for increasing values of x.
- Students will inductively derive the natural base ***e*** using $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$.
- Students will apply the natural base ***e*** in the continuous compounding formula $B = Pe^{rt}$.
- Students will be able to identify $y = ax^b$ as exponential decay when x < 1.
- Students will be able to identify $y = ax^b$ as exponential growth when x > 1.
- Students will model a geometric series of the type $\sum_{b=0}^{n-1} ax^b$.
- Students will graph exponential functions of the type $y = ax^b$.
- Students will analyze rational function behavior and limits of the form $\lim_{x\to\infty}\frac{ax^n \pm b}{cx^m \pm d}$ where n = m, n >m, and n< m.
- Students will compute Annual Percentage Yield (APY) where $APY = \left(1+\frac{r}{n}\right)^n - 1$, given the Annual Percentage Rate (APR).
- Students will use the compound interest formula to derive the present value of a single deposit investment formula
$$P = \frac{B}{\left(1+\frac{r}{n}\right)^{nt}}$$
- Students will use the compound interest formula to derive the present value of a periodic deposit investment formula
$$P = \frac{B\left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)^{nt} - 1}$$
- Students will use the future value of a periodic deposit investment formula
$$B = \frac{P\left(\left(1+\frac{r}{n}\right)^{nt} - 1\right)}{\left(\frac{r}{n}\right)}$$
- Students will adapt the algebra from banking formulas for input into a spreadsheet.

Unit 2: Investing

Students are introduced to basic business organization terminology in order to read, interpret, chart and algebraically model stock ownership and transaction data. Statistical analysis plays a very important role in the modeling of a business. Using linear, quadratic, and regression equations in that process assists students in getting a complete picture of supply, demand, expense, revenue, and profit as they model the production of a new product.

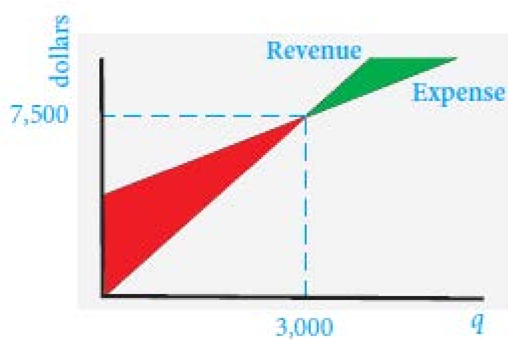
Mathematics Topics

- Algebraic ratios and proportions
- Algebraic representations of percent increase and decrease
- Pictorial representations of data
- Scatterplots
- Operations with functions
- Function domains
- Function evaluation
- Linear and quadratic functions to model situations
- Rational functions
- Systems of equations (linear/linear and linear/quadratic)
- Systems of inequalities
- Regression equations
- Extrapolation and interpolation
- Pearson Product-Moment Correlation Coefficient
- Axis of symmetry, roots, intercepts and concavity of parabolas
- Quadratic formula
- Absolute and relative extrema
- Explanatory, response, and lurking variables
- Causation vs. correlation for bivariate data
- Transitive Property of Dependence
- Zero Net Difference

Mathematical Learning Goals

- Students will construct, use, and interpret algebraic ratios and proportions.
- Given ratios of the form $r_1 : r_2 : ... : r_{n-1} : r_n$ and a total T, students will write and solve the equation $r_1x + r_2x + ... + r_{n-1}x + r_nx = T$ and determine the amount associated with each ratio.
- Students will determine, use, and interpret percent increase/decrease of monetary amounts.
- Students will determine, use, and interpret percent net change of monetary amounts.
- Students will constructing and interpret pictorial representations of data.

- Given a set of *n* data points, , students will calculate and interpret *d*-day simple moving averages by applying the Arithmetic Average Formula and the Subtraction/Addition Method.
- In any *a-for-b* stock split, where *P* represent the pre-split price per share, students will calculate the post-split price per share using
- In any *a-for-b* stock split, where *D* represent the pre-split number of shares, students will calculate the post-split number of shares using
- Students will calculate the stock yield percentage using the formula , where A represents the annual dividend per share and C represents the current price per share.
- Students will construct and interpret scatterplots .
- Students will identify form, direction, and strength from a scatterplot.
- Students will perform operations with functions.
- Students will evaluate functions and use them to model situations.
- Students will translate verbal situations into algebraic linear functions.
- Students will translate verbal situations into quadratic functions.
- Students will create rational functions of the form $f(x) = \frac{mx+b}{x}$.
- Students will translate verbal situations into linear and quadratic inequalities.
- Students will solve linear systems of equations and inequalities such as:



- Students will solve systems of linear equations and inequalities in two variables.
- Students will identify domains for which f(x) > g(x), f(x) = g(x), and f(x) < g(x).
- Students will find interpret, and graph linear regression equations.
- Students will determine domains for which prediction using a regression line is considered extrapolating or interpolating.
- Students will find and interpret the Pearson Product-Moment Coefficient of Correlation
- Students will find the axis of symmetry $x = \frac{-b}{2a}$, vertex $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$, roots, and the concavity of parabolic curves.
- Students will use the quadratic formula $if \ ax^2 + bx + c = 0 \ then \ x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$.

- Students will find and interpret quadratic regression equations.
- Students will solve linear-quadratic systems of equations and inequalities such as:

- Students will find absolute and relative extrema.
- Students will delineate Causation vs. correlation for bivariate data.
- Students will identify explanatory and response variables.
- Students will identify and diagram lurking variables.
- Students will use the transitive property of dependence.
- Students will determine the zero net difference.
- Students will write algebraic formulas for use in spreadsheets.
- Students will use, interpret and evaluate rational expressions.
- Students will use, interpret and evaluate algebraic fractions, ratios, and proportions.

Unit 3: Employment and Income Taxes

Many Internal Revenue Service and Social Security Administration regulations can be modeled by using linear and polygonal functions that have different slopes over different domains. Line-by-line instructions for IRS forms can also be algebraically symbolized.

Mathematics Topics

- Point-slope form of linear equations
- Jump discontinuities
- Continuous functions with cusps
- Slope
- Compound inequality notation
- Piecewise functions
- Interval notation
- Percent increase and decrease
- Data analysis
- Algebraic modeling

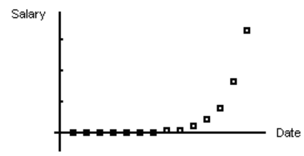
Mathematics Learning Goals

- Students will identify continuous and discontinuous functions by their graphs.

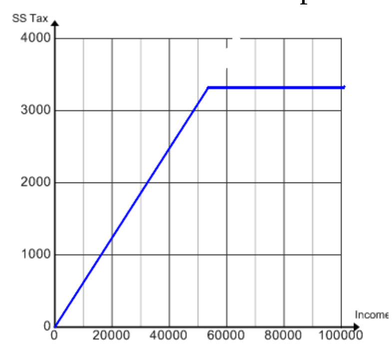
- Students will interpret jump discontinuities.
- Students will determine and interpret domains of piecewise functions of the forms

$$r(x)=\begin{cases}29.95 \text{ if x is an integer and } x \le 2\\ 29.95+14(x-2) \text{ if x is an integer and } x>2\end{cases} \qquad c(x)=\begin{cases}0.20x \text{ when } 0\le x<750\\ 0.22x \text{ when } 750\le x\le 1{,}000\\ 0.25x \text{ when } x>1{,}000\end{cases}$$

- Students will graph exponential pay schedules such as



- Students will graph piecewise functions with cusps such as



- Students will compute measures of central tendency and rational functions such as

$$a(x)=\frac{40r+1.5tr}{t+r}.$$

- Students will use geometric sequences such as $a_n = xr^n$ with common ratio r.
- Students will express percent increases and decreases as rational functions.
- Introducing point-slope form $y-y_1=m(x-x_1)$ and converting it to slope-intercept form $y=mx+b$.
- Students will graph continuous polygonal functions with multiple slopes and cusps.

- Students will translate verbal expressions into literal rational, exponential, and linear equations.
- Students will express domains using compound inequality notation of the form $t \ge t_1$ *and* $t < t_2$.

- Students will express domains using compound inequality notation of the form $t > t_1$ *and* $t \le t_2$, interval notation of the form $t_1 < x \le t_2$, and tax schedule notation of the form "over t_1 but not over t_2."
- Students will model a tax bracket, given a compound inequality statement, and model a tax bracket to determine the tax using a linear equation of the form. $y = a + p(x - t_1)$ where y is the tax, a is the base tax, p is the tax percentage expressed as a decimal, (t_1 is the lower boundary of the domain, and x is the taxable income).
- Students will convert point-slope form to slope-intercept form of a linear equation.
- Students will write equations in point-slope form.
- Students will model algebraically a tax schedule of the form:

Schedule Y-1— If your filing status is **Married filing jointly** or **Qualifying widow(er)**

If your taxable income is: Over—	But not over—	The tax is:	of the amount over—
$0	$16,050	.... 10%	$0
16,050	65,100	$1,605.00 + 15%	16,050
65,100	131,450	8,962.50 + 25%	65,100
131,450	200,300	25,550.00 + 28%	131,450
200,300	357,700	44,828.00 + 33%	200,300
357,700	....	96,770.00 + 35%	357,700

- Students will create and interpret piecewise functions of the form

$$f(x) = \begin{cases} 0.10x & 0 < x \le 16{,}050 \\ 1{,}605 + 0.15(x - 16{,}050) & 1{,}605 < x \le 65{,}100 \\ 8{,}962.50 + 0.25(x - 65{,}100) & 65{,}100 < x \le 131{,}450 \\ 25{,}550 + 0.28(x - 123{,}700) & 131{,}450 < x \le 200{,}300 \\ 44{,}828 + 0.33(x - 200{,}300) & 200{,}300 < x \le 357{,}700 \\ 96{,}770 + 0.35(x - 357{,}700) & x > 357{,}700 \end{cases}$$

where f(x) represents the tax liability function for taxpayers using a given tax schedule with taxable incomes on a given domain

- Students will graph piecewise functions of the form

$$f(x) = \begin{cases} y = 0.10x & 0 < x \le 16{,}050 \\ y = 0.15x - 802.5 & 16{,}050 < x \le 65{,}400 \\ y = 0.25x - 7{,}312.5 & 65{,}100 < x \le 131{,}450 \end{cases}$$

- Students will determine the cusps of piecewise functions from the function notation.
- Students will interpret the graphs, slopes, and cusps of continuous polygonal functions with multiple slopes and cusps.
- Students will adapt all algebraic formulas in the unit for use in spreadsheets.

Unit 4: Automobile Ownership

Various functions, their graphs, and data analysis can be instrumental in the responsible purchase and operation of an automobile.

Mathematics Topics

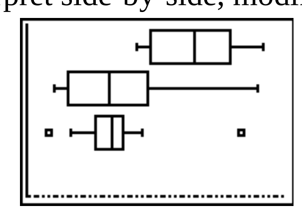
- Exponential/linear systems of equations
- Irrational functions
- Quadratic functions
- Arc length
- Piecewise functions
- Graphs of piecewise functions
- Systems of linear equations
- Frequency distributions
- Stem-and leaf plots
- Modified box-and-whisker plots
- Measures of dispersion
- Quartiles
- Interquartile range
- Outliers of a frequency distribution

Mathematics Learning Goals

- Students will model exponential depreciation as $y = Px^b$ .where P is the purchase price and x < 1, and compare the depreciation to an increasing linear expense function.
- Students will transform raw data into a frequency distribution.
- Students will create and interpret stem and leaf plots and side-by-side steam plots such as

9 8 1 1 1 1	87	1 2 2
3 2	88	2 4 6 7
7 5 4	89	1 3
7 6 6 6 6 5	90	2 7 7 7

- Students will create and interpret side-by-side, modified box and whisker plots as shown:

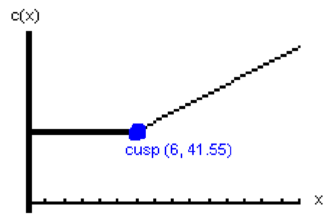


- Students will compute measures of dispersion $R = x_H - x_L$ and $IQR = Q_3 - Q_1$.
- Students will compute Q_1, Q_2, Q_3, and Q_4 manually and with the graphing calculator.
- Students will compute boundaries for outliers using the expressions $Q_1 - 1.5(IQR)$ and $Q_3 + 1.5(IQR)$.
- Students will compute and interpret percentiles.

- Students will compute measures of central tendency $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$, median and mode.
- Students will create and interpret piecewise (split) functions of the form

$$c(x) = \begin{cases} 38 \text{ when } x \leq 4 \\ 38 + 6.25(x-4) \text{ when } x > 4 \end{cases}$$

- Students will determine the domains of a piecewise function from verbal situations.
- Students will graph piecewise functions using mutually exclusive domains.
- Students will determine the cusp of a piecewise function at a change in slope such as



- Students will use multi-variable square root functions such as the skid length $S = \sqrt{30Dfn}$.
- Students will determine the reaction distance using the formula $RD = 0.75\left(\frac{5280s}{60^2}\right)$.
- Students will compute braking distance using the formula $BD = 5(.1s)^2$.
- Students will compute total stopping distance using the formula

$$TSD = 0.75\left(\frac{5280s}{60^2}\right) + 5(0.1s)^2.$$

- Students will compute distance, rate and time using $D = RT$, $R = \frac{D}{T}$, and $T = \frac{D}{R}$.
- Students will compute miles per gallon and distance using the formula $D = MPG(G)$.
- Students will use geometry theorems involving chords intersecting in a circle and radii perpendicular to chords to determine yaw mark arc length.
- Students will find the radius $r = \frac{C^2}{8M} + \frac{M}{2}$ where C is chord length and M is middle ordinate
- Students will compute arc lengths.
- Students will use dilations D_k to transform formulas between the English Standard and Metric measurement systems.
- Students will adapt all algebraic formulas from the chapter for use in spreadsheets.

Unit 5: Consumer Credit

Becoming familiar with credit terminology and regulations is critical in making wise credit decisions. Credit comes at a price and in this unit students learn how to use mathematics to make wise credit choices that fit their needs, current financial situation, and future goals.

Mathematics Topics

- Algebraic proportions
- Linear, quadratic, cubic, and exponential equations
- Exponential growth and decay
- Regression equations
- Inverse function of an exponential equation
- Logarithms
- Summation notation

Mathematics Learning Goals

- Students will create, evaluate, interpret and solve algebraic proportions.
- Students will model situations using linear, quadratic, cubic, and exponential equations.
- Students will determine the curve of best fit using linear, quadratic, or cubic regression equations.
- Students will create, use, and interpret exponential growth and decay equations that model given situations.
- Students will apply an exponential equation in the form of the monthly payment formula

$$M = \frac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{12t}}{\left(1+\frac{r}{12}\right)^{12t}-1}.$$

- Students will use the slope-intercept form y=Mx+b where $M = \frac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{12t}}{\left(1+\frac{r}{12}\right)^{12t}-1}$

- Students will use the formula where FC = finance charge and R = retail price

$$FC = \left[\frac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{12t}}{\left(1+\frac{r}{12}\right)^{12t}-1}\right]x+b-R$$

- Students will use inverse functions to create the natural logarithm function $y=\ln x$ as $y=\log_e x$ and as the inverse of $y=e^x$.

- Students will solve for the exponent t in the monthly payment formula

$$t = \frac{\ln\left(\frac{M}{p}\right)-\left(\ln\left(\frac{M}{p}-\frac{r}{12}\right)\right)}{12\ln\left(1+\frac{r}{12}\right)}$$

- Students will interpret and use summation notation to model the average daily balance

$$\sum_{i=1}^{n} \frac{d_n}{n}$$

- Students will calculate the finance charge using the formula $FC = \left(\sum_{i=1}^{n} \frac{d_n}{n} \right) \frac{APR}{12}$
- Students will create and use algebraic formulas and apply them for use in spreadsheets.

Unit 6: Independent Living

In this unit, students work their way through the mathematics that models moving, renting, and purchasing a place to live. They also explore the geometric demands of floor plans and design, and discover the relationship between area and probability.

Mathematics Topics

- The apothem of a regular polygon
- Area of a regular polygon
- Areas of shaded regions
- Rational functions
- The Monte Carlo Method
- Exponential functions
- Dilations and scale

Mathematics Learning Goals

- Students will use rational functions to compute back-end and front-end ratios of the form

$$b = \frac{m + p/12 + h/3 + c + d}{a/12} \textbf{ and } f = \frac{m + p/12 + h/12}{x/12} ..$$

- Students will make computations based on the monthly payment formula

$$M = \frac{\left(P\left(\frac{r}{12}\right)\left(1 + \frac{r}{12}\right)^{12t} \right)}{\left(\left(1 + \frac{r}{12}\right)^{12t} - 1 \right)}$$

- Students will compute mortgage interest where C is original cost and

$$I = \frac{\left(P\left(\frac{r}{12}\right)\left(1 + \frac{r}{12}\right)^{12t} \right)}{\left(\left(1 + \frac{r}{12}\right)^{12t} - 1 \right)} - C$$

- Students will use the apothem to derive the formula for the area of a regular polygon

$$A = \frac{1}{2}ap$$

- Students will use probability to find the area of irregular plane region (The Monte Carlo Method)

$$\frac{\text{number of points inside region}}{\text{number of random points generated}} = \frac{\text{area of irregular region}}{\text{area of framing rectangle}}$$

- Students will use factors of dilations to draw to scale.
- Students will compute areas of irregular and shaded regions.
- Students will use rational functions to compute BTU's, such as $\text{BTU rating} \approx \frac{while}{60}$.
- Students will solve scale problems using proportions.
- Students will use literal equations to create multi-variable tax assessment equations.
- Students will us exponential equations to model rent increases such as $R = A\left(1+\frac{B}{100}\right)^{D-1}$.

- Students will model rent increases using exponential regression functions.
- Students will read and interpret data.
- Students will use the future value of a periodic deposit formula $B = \frac{P\left(\left(1+\frac{r}{n}\right)^{nt}-1\right)}{\left(\frac{r}{n}\right)}$ to make comparisons to mortgage payments and increasing resale value of a home.
- Students will adapt all algebraic formulas for use in spreadsheets.
- Students will translate verbal expressions into literal equations.

Unit 7: Retirement Planning and Budgeting

The focus of this unit is on the mathematics of fiscal plans that workers can make years ahead of their retirement date. This involves a detailed study of retirement savings plans, both personal and federal, employee pension programs, and life insurance. Additionally, students are asked to call upon the knowledge acquired in all of the preceding units in order to create and chart a responsible personal budget plan, to mathematically analyze cash flow, and to determine net worth.

Mathematics Topics

- Expected value of a probability distribution
- Greatest Integer function
- Sectors and central angles
- Exponential Equations
- Rational expressions as combinations of rational and polynomial expressions
- Piecewise Greatest Integer Function
- Systems of linear and piecewise functions
- Domains, constants, coefficients, dependent and independent variable

Mathematics Learning Goals

- Students will use the future value of a periodic investment formula of the form

$$B = \frac{P\left(\left(1+\frac{r}{n}\right)^{nt}-1\right)}{\frac{r}{n}}$$

  to predict balances after t years when given a periodic deposit amount, an investment return rate, and compounding information.
- Students will use the present value of a periodic investment formula of the form

$$P = \frac{B\left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)^{nt}-1}$$

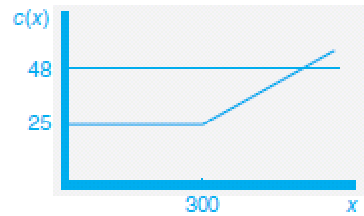
  to determine the principal when given a future value, a time in years, an investment return rate, and compounding information.
- Students will write rational expressions as a combination of rational and polynomial expressions.
- Students will use inequalities to define domains when creating algebraic expressions.
- Students will analyze the effect that a change in multipliers has to the value of an algebraic expression.
- Students will write rational expressions to represent increase over time.

- Students will use and interpret the greatest integer function of the form $[x]$.
- Students will determine and interpret the expected value of a probability distribution where the expected value is of the form $\sum_{i=1}^{n} x_i f(x_i)$.
- Students will read and interpret data presented in multiple formats.

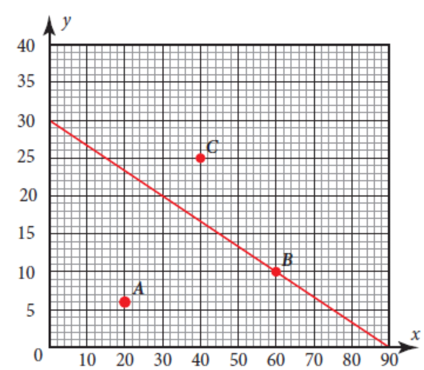
- Students will create interpret, and graph greatest integer functions of the form $y=[x-a]$.
- Students will create, interpret, and graph greatest integer functions of the form $y=[x-a]+1$.
- Students will understand the algebraic and contextual differences between $y=[x-a]$ and $y=[x-a]+1$.
- Students will incorporate the greatest integer function into a piecewise function of the form

$$c(x) = \begin{cases} a \text{ when } x \le b \\ a + c(x-d) \text{ when } x > b \text{ and x is an integer} \\ a + c([x-d]+1) \text{ when } x > b \text{ and x is not an integer} \end{cases}$$

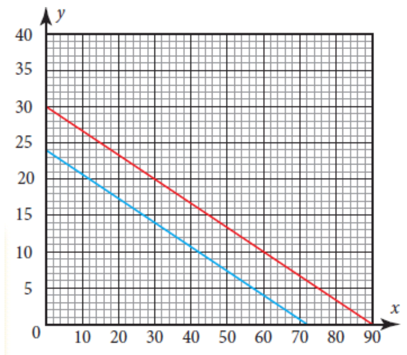
- Students will evaluate a piecewise function that includes a greatest integer function for various values on the domain of the piecewise function.
- Students will create, interpret, and graph a system of a linear and a piecewise function and determining the point of intersection as shown in the following graph:



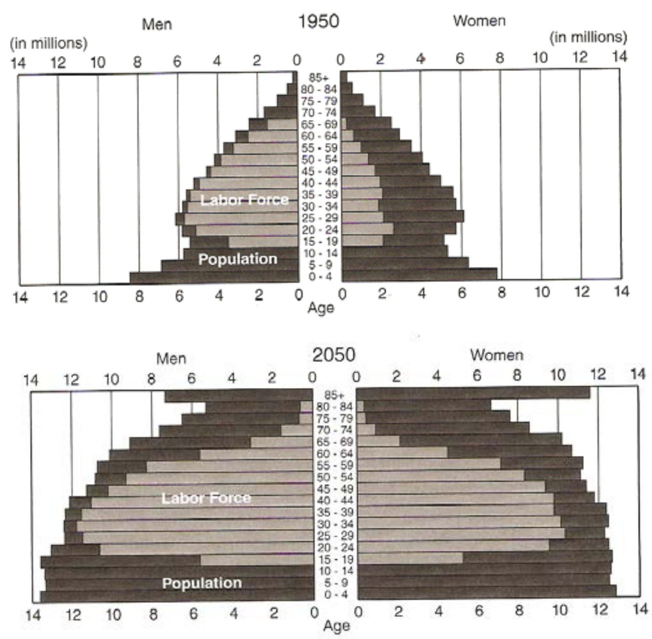
- Students will use sectors and central angles of a circle to depict proportional categories on a pie chart when given categorical information.
- Students will create and interpret budget line equations of the type $C_x x + C_y y = B$ where C_x represents the cost of the first of two items and C_y represents the cost of the second of two items, x and y represent quantities under consideration and B represents an amount budgeted.
- Students will interpret points on a budget line graphs in the context of their relationship to the budget line as shown in the following display:



- Students will compare budget line graphs and interpret them as transformations in the plane as shown here:



- Students will use inequalities to interpret regions and points in the plane in relation to a budget line graph.
- Students will use multiple representations to chart data such as



Bureau of Labor Statistics

Advanced Algebra with Financial Applications
KEY ASSIGNMENTS

The Key Assignments presented in this section are well-aligned with the CCSS Standards for Mathematical Practice. The assignments are all verbal problem solving activities that relate to the unit being studied. Students must represent the verbal situation symbolically, manipulate those symbols to arrive at an answer, and then interpret that answer in the context of the problem. This offers students opportunities to make sense of quantities and their relationships within those problem-solving settings through multiple representations. Students can approach, access, and deconstruct the necessary mathematics using handheld graphing utilities, manipulatives, spreadsheets, and/or software. The assignments throughout this course require students to attend to precision in their responses both in the computational and algebraic fluency required to arrive at those answers and in the units used to contextualize the answers.

The prevalence of mathematical modeling assignments allows students to practice seeking out mathematical structure in what may seem to them to be an unstructured situation. Identifying and exploiting the structure leads students to a richer understanding of the themes and regularities that are present in the real world. Students make tables, find patterns, and offer conjectures based on the patterns. This form of inductive reasoning is a cornerstone of mathematical thinking. The assignments and other course-related activities optimize students' exposure to extrapolating what they have learned to routine and non-routine mathematically-dependent situations they encounter in their futures.

Most assignments require the student to prepare a presentation on their finished work. This can be a PowerPoint show, a webinar, a poster presentation, or a presentation using transparencies. The student audience gets to critique the presentation, ask questions, and make comments, in a firmly established, constructive, positive "safe" zone. The presentation is graded, and the quality of student critiques and comments can also be graded.

Unit 1: Banking

Key Assignment 1.1: How Interest Method Affects Monetary Growth

Mathematics: Simple interest, compound interest
Mathematics Learning Goals: To determine how increased compounding affects growth.

Students are first introduced to the meaning of compounding numerically via mathematical iteration. Before embarking on a rigorous study of limits and compound interest algebraic formulas, students are asked "How much would $1,000 grow to, in one year, at 100% interest compounded continuously?" The 100% interest and continuous compounding often leads them to guess much higher than the actual amount. Their guesses are recorded, and a statistical analysis of their guesses is made. Outliers are carefully noted. The findings of this activity are scrutinized after students complete Key Assignment 3.

Key Assignment 1.2: Deriving the Compound Interest Formula

Mathematics: Inductive reasoning, exponential functions, rational functions
Mathematics Learning Goals: To use patterns and induction to generate for selected forms of compounding and adapt them to monthly, weekly, daily, and hourly compounding.

Students will compute interest for each interest period over a semi-annual and quarterly compounded account for a given balance and interest rate. They will derive the general algebraic formulas for these two types of compounding. They will then look for patterns in the semi-annual and quarterly compound interest formulas to inductively conjecture about the general formula for compounding. They will then find formulas for monthly, weekly, daily and hourly compounding, and compute and compare the interest earned over one year for these accounts.

Key Assignment 1.3: Using Limits to Derive the Natural Base e

Mathematics: Rational functions, exponential functions
Mathematics Learning Goals: To use substitution and patterns to generate a series that approaches e as x approaches infinity.

Students will be introduced to the notion of limits and limit notation and apply it to the compound interest formulas previously derived. They will increase the number of compoundings by first computing interest when the compounding period is every minute, and then every second, for a given balance and interest rate. They will then let the number of compoundings 'n' approach infinity to see what happens to the annual interest as the number of compoundings approaches infinity. They will analyze the compound interest formula without the balance, and explain the "battle" between the base and the exponent of the expression $\left(1+\frac{1}{n}\right)^n$ as $n \to \infty$.

Key Assignment 1.4: Future Value and College Costs

Mathematics: Rational functions, regression
Mathematics Learning Goals: To estimate the cost of a college education in 18 years and determine how much needs to be saved each month to have the costs covered by the 18^{th} year.

Students pick a college and find out the cost of tuition, room and board (if necessary) and fees over the past ten years. They set up a regression line or curve of best fit. They then predict the cost of a college education in 18 years (as if they just had a child and were trying to save for college). They then use the prevailing interest rate and the future value formula to determine the monthly periodic deposit that would be necessary to have the full college cost saved by the child's 18^{th} birthday. They then do the problem with interest rates slightly higher than the prevailing rate.

Unit 2: Investing

Key Assignment 2.1:Charting a Corporate Stock

Mathematics: Data Analysis, regression, prediction, modeling, graphical interpretation
Mathematics Learning Goals: The goal of this assignment is to have students use mathematical modeling to chart and interpret stock market trends over a 15-day period. They will make trend predictions based on simple moving average crossover analysis as well as regression models.

Each student selects a corporation traded on the New York Stock Exchange. They produce a background paper, PowerPoint presentation or poster board display on that corporation. Students chart the open, close, high, low and volume data for 15 consecutive trading days. They graph the data using two different formats and then discuss trends that the data shows. They will also calculate three different cluster-lengths of moving averages and, using those clusters, they will create superimposed line graphs. Students discuss trading implications based upon stated domains of graph pairs before and after any intersection points. Finally, they determine the closing price curve of best fit using regression analysis. They must state the regression equation and support why their stated curve best fits the data of closing prices. Students will then use the curve of best fit to predict a closing price on the 16th trading day. They compare that predicted price with the actual closing price on the 16th day and find a percent error.

Key Assignment 2.2: Mathematically Modeling A Business

Mathematics: Linear and quadratic functions, linear/linear Systems, linear/quadratic systems, regression analysis
Mathematics Learning Goals: To have students create linear and quadratic models for a start-up business. They will graph and interpret systems of these regression and modeling equations in order to explore the relationship between and among expense, demand, price, revenue and profit.

Students are given a market research scenario for a new product, attained from a focus group questionnaire. The research contains a list of ordered pairs in the form (p,q) where p is a potential price and q is the quantity of the product that the focus group member would purchase if it was set at that price. Using these ordered pairs, students construct a scatterplot, determine the correlation coefficient, and identify a linear regression equation in which q is the independent variable and p is the dependent variable. Then, given information about expenses, they are to set up a linear expense function in terms of the quantity demanded. The quadratic revenue and profit equations are determined and graphed on the same axes with the expense function. Students identify and interpret the breakeven points, the coordinates of the maximum point on the revenue graph, the coordinates of the maximum point on the profit graph, and the price at which the product should be sold in order to maximize profit. Finally, students are told the initial price per share for the company’s stock and asked to determine the number of shares that must be sold in order to have enough money to start this business.

Unit 3: Employment and Income Taxes

Key Assignment 3.1: Creating the Tax Worksheet

Mathematics: Domains, piecewise functions, linear functions and graphs, point-slope form, slope-intercept form, graphs with cusps.
Mathematics Learning Goals: To derive the slope-intercept form used on the IRS tax worksheet by translating tax tables into piecewise functions.

The tax tables give taxpayers a function in which the independent variable is the taxable income and the dependent variable is the tax. It is convoluted and has confused taxpayers for years. Within the last decade, the IRS created a worksheet that uses the slope-intercept form of the equations of a line to simplify calculations for the taxpayer. In this Key Assignment, students interpret the IRS Schedule, express the domains using compound inequality notation, and create the piecewise function that models the IRS intentions. They then covert this function, which is a translated version of point-slope form, into the slope-intercept form to create the tax worksheet.

Key Assignment 3.2: Graphing the FICA Tax Function

Mathematics: Piecewise functions, slope, cusps, linear equations
Mathematics Learning Goals: To use graphs to compare the FICA tax longitudinally over a prescribed number of years.

Students look up the FICA tax percents, and maximum taxable incomes to create piecewise functions for each of the last six years. They compute the maximum FICA tax, and graph all six years on the same axes, and use the graph to write a paragraph on what has happened to FICA taxes over those years. They discuss the significance of the coordinates of the cusp. They do the same for the tax years 1981-86, and compare the last six years to the years 1981-1986. The assignment is replicated using the Medicare tax percent.

Unit 4: Automobile Ownership

Key Assignment 4.1: Using Statistics to Negotiate Auto Transactions

Mathematics: Bivariate data, **c**orrelation, regression, mean, median, mode, quartiles, interquartile range, outliers, modified box-and-whisker plots, stem-and-leaf plots, frequency distributions, scatterplots.
Mathematics Learning Goals: To use measures of central tendency and measures of dispersion to mathematically negotiate the buying and/or selling of an automobile.

Students choose a make, model and year for an automobile. They use the Internet and newspaper classified ads to find 10-20 of those cars for sale. They get the price of the car and the mileage it has. They construct modified box-and-whisker plots and describe the frequency distribution. They pair each car's price with its mileage to create a scatterplot. They classify the association as positive or negative. They find the regression line and correlation coefficient and interpret the relationship as strong, moderate or weak, and discuss its linearity. Their results are presented to the class via PowerPoint presentation or poster presentation.

Key Assignment 4.2: Automobile Cost and Depreciation

Mathematics: Exponential regression, **g**raphing linear and exponential functions, rational functions, linear/exponential systems, systems of linear equations, slope-intercept form.
Mathematics Learning Goals: To use graphing techniques to compare the value of a car to the expense of purchasing it throughout its lifetime.

Using the monthly payment rational function, students graph the cost C of purchasing a new car, using the down payment as the y-intercept, and the monthly payment as the slope. They then investigate three types of depreciation: straight-line, exponential, and historical bath tub graphs. They graph the cost and depreciation functions on the same set of axes to find the month at which the total cost C of owning the car surpasses its value V as it depreciates. They identify and interpret the domains on which $C > V$ and $C < V$.

Key Assignment 4.3: The Physics of Driving

Mathematics: Quadratic equations, radical functions, arc length, geometry of the circle.
Mathematics Learning Goals: To use the mathematics listed to determine braking distances and to gather data from accidents scenes.

Students use formulas to determine reaction distance, braking distance, and figure out the speed a car was going based on its skid marks. The braking-distance formula is a quadratic function, with speed as the independent variable. The skid speed formula is an irrational function that has three independent variables. Students also use the geometry of the circle to compute the radius of a given yaw mark, which is a curved skid mark, and use the radius and friction factor to find the speed the car was going when it began to skid. The students then prepare a PowerPoint or poster presentation for the driver's education class in their school.

Unit 5: Consumer Credit

Key Assignment 5.1: Can I Afford This Loan?

Mathematics: Exponential functions, logarithmic functions, system of exponential and linear functions, modeling, graphical interpretation
Mathematics Learning Goals: To use three modalities to determine the affordability of a loan: exponential formula evaluation, logarithmic formula evaluation, and interpreting an exponential/linear system. To use technology (graphing utility and/or spreadsheet) to make the determinations required and justify their responses.

Students are given a scenario in which a family must make a decision about the affordability of a loan based on the principal, the loan-length, the APR and the maximum affordable monthly payment the family is able to make towards loan debt reduction. Students determine the affordability of the loan in three different ways: using the monthly payment function, interpreting the graphs of the system of equations defined by the exponential monthly payment function and the linear maximum affordable monthly payment, and using the logarithmic loan length function. They are then asked to construct two spreadsheets: a monthly payment spreadsheet that charts the monthly payment as loan length time varies from 1 to 20 years, and a loan length spreadsheet that charts time as monthly payments vary from \$100 to \$1000. Finally, students must write up a summary analysis for this situation explaining how the algebraic modeling by the spreadsheet formulas supports their prior work.

Key Assignment 5.2: Mathematically Modeling a Credit Card Statement

Mathematics: Algebraic modeling and spreadsheet formula creation
Mathematics Learning Goals: To algebraically model a month of activity on a person's credit card.

Students create a 21-day credit calendar that depicts algebraic representations of daily balances based upon an opening balance of Y dollars, an X-dollar purchased on the 8th day, a Z dollar payment on the 13th day, and a W-dollar purchased on the 20th day. Using these representations from the calendar, they write algebraic expressions for the sum of the daily balances, the average daily balance, and the finance charge for this 21-day period given that the APR on this credit card is P%. Students then create a spreadsheet that models the situation described above and test their spreadsheet for a given data set.

Unit 6: Independent Living

Key Assignment 6.1: Areas of Irregular Plane Figures

Mathematics: Probability, ratios, random integers, graphing, random number table
Mathematics Learning Goals: To use the Monte Carlo method to find the area of any regular or irregular plane figure.

Students superimpose a grid on an irregular plane figure that is part of a landscape design. They outline the irregular figure with a rectangle and use a random number generator from a calculator, or a random number table, to generate 500 points, which they plot on their rectangular grid. As they plot each point, they note if it is inside or outside of the irregular region. They find the percent of random points that landed in the irregular region and take that percent of the area of the enclosing rectangle to approximate the area of the irregular region.

Key Assignment 6.2: Areas of Shaded Regions

Mathematics: Area formulas
Mathematics Learning Goals: To determine areas of plane figures that have sections removed from them.

As part of a unit on floor plans and interior design, students compute areas of floors to find the cost of new flooring. They also compute the cost of paint by taking the areas of the walls and subtracting window and door areas. They employ the area of a circle, square, triangle, rectangle, trapezoid, and parallelogram, and create a poster display on what a specific room cost to redo.

Key Assignment 6.3: The Apothem and the Area of a Regular Polygon

Mathematics: Inscribed circles, area of a triangle, perimeter, congruence.
Mathematics Learning Goals: To derive a formula for the area of any regular polygon.

Students use the area of a triangle to find the area of a regular polygon. They divide a regular polygon into triangles, by connecting the center to each vertex. They draw in the altitude, which is renamed the apothem, and find the area of the triangle. They discuss the congruence of the n triangles formed in the regular n-gon, and multiply to find the area of the polygon. They then model this algebraically, and use the commutative property of multiplication to derive the formula that the area is half the product of the apothem and the perimeter of the regular polygon.

Key Assignment 6.4: How Increased Payments Affect Mortgages

Mathematics: Rational functions
Mathematics Learning Goals: To determine the reduction in interest that extra mortgage payments result in.

Students use the monthly payment formula to compute the monthly payment for a hypothetical mortgage amount over 15 and 30 years. They compute the total payments, based on 12 monthly payments each year, and the total interest for the entire loan. They then use a mortgage calculator to assume an extra, 13th payment is made each year, so payments are made once every 4 weeks instead of once each month. They compute the interest and new total repayment period and compare the total interest to the original conventional mortgage to see the savings in total years and interest.

Unit 7: Retirement and Budgeting

Key Assignment 7.1: How Do Life Insurance Companies Earn a Profit?

Mathematics: Expected value, random variables, probability distributions
Mathematics Learning Goals: To use probability distributions and mortality tables to compute the profit earned on a five-year term life insurance policy.

Students use the probability inherent in mortality tables and life insurance annual premiums to compute the expected profit for a life insurance company’s term policy. They create probability distributions for the random variable profit and compute expected profit by summing the products of the individual profits and probabilities for each year of the policy. They compute the minimum annual premium the company must charge to earn a profit.

Key Assignment 7.2: Planning For Retirement

Mathematics: Exponential equations, expected value, data analysis, modeling and predicting
Mathematics Learning Goals: To apply prior knowledge from the banking unit to make decisions about the feasibility of a retirement plan.

Students are given financial information about a prospective retiree and asked to act as a financial retirement planner. The prospective retiree has also supplied the planner with desired monetary goals in retirement. Based upon information about savings plans, social security benefits, pensions, and life insurance policies, and using formulas learned in this unit, the planner is to write up a financial plan for the prospective retiree that includes at least two ways of meeting the goals and has mathematical justification for the recommendations made.

Key Assignment 7.3: Cash Flow, Net Worth and Debt Reduction

Mathematics: Algebraic ratios, modeling, linear equations
Matheamtics Learning Goals: To create a spreadsheet that calculates cash flow, net worth, and debt to income ratio.

Students are given a budget spreadsheet that contains the headings of income, fixed expenses, variable expenses, and non-monthly expenses. There are sub-headings under each of these listing specific categories relating to the heading. Students are given a full accounting of a person's financial status and asked to build a spreadsheet that calculates that person's cash flow. In addition, the students are given information about the person's assets and liabilities and are asked to add it to the spreadsheet and determine the net worth. Finally, based upon the calculation of the debt-to-income ratio, students are asked to develop a debt reduction plan for the individual if necessary.

Key Assignment 7.4 Budget Line Equations

Mathematics: Linear equations, domain, range, constraints, modeling,
Mathematics Learning Goals: To construct and interpret a graphical representation of a particular aspect of a budget.

A budget line graph allows the user to interpret many combinations of product usage based upon given constraints. The interpretation of the combinations allows the user to make decisions about affordability. Students are given information about a particular aspect of a personal budget. This data contains prices and budgeting constraints. Students are asked to construct a budget line equation of the form where and are costs related to two budgeted items, x and y, and B is the budgeted amount. They then examine the regions above, on, and below the budget line to identify points representing affordability data. Students make recommendations for this budget item based upon the interpretation of the budget line graph.

Advanced Algebra with Financial Applications
INSTRUCTIONAL METHODS AND STRATEGIES

The instructional strategies used throughout this course are varied, targeted, and rooted in the CCSS Standards for Mathematical Practice. Just as the Standards are interrelated, the methods used in this course are. Together, the practices referenced in this section serve to build mathematical confidence, interest and strength.

The Advanced Algebra with Financial Applications program's instructional strategies cover these basic umbrellas:

- Motivational Unit Openers
- Essential Questions
- Reading
- Discussion/interaction
- Presentation of model problems
- Extensions and problem solving
- Differentiation of instruction
- Experiential learning
- Use of technology

The **motivational unit openers** are real-life problems that need to be solved mathematically. Students realize that they "need to know" this material, as they will be encountering financial matters every days of their adult lives. Financial situations are inherently natural motivators. Since all of the problems in the course are real-world applications, lessons must integrate **reading and discussion** on a daily basis. An **essential question**, written on the board each day, serves as a focal point as algebraic symbols are used to represent the situation. These applications are all embedded in prose, so every new topic begins with a reading passage that acts as a springboard to a full-class discussion. This lively interactive feature of every lesson sets a constructive, motivating stage for the mathematics that follows.

The direct instruction/lecture component is highlighted by the investigation of **model problems** on each skill covered. After each model problem, students look for structure and regularity and try to apply it in a situation rooted in the model problem just completed. This gives the students a chance to see if they understood the new concept before moving on to a deeper problem for which the previous problem was an entry condition. Students are then asked to extend their understanding by looking for patterns and **extending** previously-used strategies. The applications at the end of each section give students a chance to practice as part of their classwork and homework. The program spirals previously-learned material on a daily basis. The sequential nature of the introduction of each new skill, followed by immediate practice, allows students to monitor their progress often. Class notes include vocabulary and financial explanatory material as well as mathematical procedures.

The model problems and applications generally graduate in difficulty level, allowing the teacher to **differentiate instruction**. Since abstract reasoning can be difficult for many students, the instructions are graduated so students can grasp the higher level skills by meeting them step-by-step. This strategy allows student and teacher to identify the exact juncture at which the student is having difficulty. This makes diagnostics and intervention more pointed.

There is much opportunity for **experiential learning**. Projects require the students to get out in the field and meet with brokers, bankers, local businesses, etc. Guest speakers at several junctures bring the outside world right into the classroom. Students act as moderators and compile questions for the guest speaker. For some projects, data is gathered and statistically analyzed. Students present their work to the class, and they field questions and comments from their classmates.

Technology plays a key role in the development of Advanced Algebra with Financial Applications topics. The graphing calculator is a daily tool, and its algebraic and graphing features are extensively used. Spreadsheets appear in every unit so students can model situations using algebra and technology.

Advanced Algebra with Financial Applications
ASSESSMENT METHODS

A variety of formative and summative assessment methods are used throughout Advanced Algebra with Financial Applications in order to assess student learning. The assessments are aligned with the course purpose and the instructional strategies used, and with the Common Core Standards for the development of mathematically proficient students. In the activities listed below, students are offered assessment opportunities to address mathematics as a sense-making tool, problem solve, reason, construct arguments, offer mathematics-justified critiques of arguments, , model, use appropriate tools, attend to precision, look for and make use of structure, and look for and express regularity in repeated reasoning. The assessment grading percentages contributing to the student's quarter course grade are offered in parentheses next to the assessment name.

FORMATIVE ASSESSMENTS (30%)

CLASS PARTICIPATION (15%)

- **Do Now Activities** are assessments that can be used as a vehicle for the teacher to determine whether students have acquired skills, strategies, and content necessary for subsequent work in a topic. This diagnostic feature allows the teacher to adjust the lesson accordingly, if entry conditions are not fully met.

- **Check Your Understanding** problems are offered to students immediately after the teacher has introduced a new concept or procedure. These problems offer students and teacher alike an immediate assessment opportunity that is confined to the single new skill just addressed. The teacher can adjust the lesson to follow based upon review of these problems.

- **Extend Your Understanding** problems are more advanced problems that use the concepts and procedures just learned and take them to another level. These can be offered to all students or differentiated for selected students depending on the nature of the problems.

- **Ticket to Leave Activities** are ungraded activities that offer the teacher an opportunity to determine the level of understanding students acquired on the skills, strategies, and content of the day's lesson. These activities can be used by the teacher to adjust the following day's lesson.

- **Direct and Indirect Teacher Questions** are immediate formative methods of assessing students' understanding. In-class discussion is a critical part of Advanced Algebra with Financial Applications. The teacher should initiate discussion through focused questioning.

- Through the **Exploration of Essential Questions** (one per lesson), the teacher assesses student understanding both pre-instruction and post-instruction. The essential question is offered to the students at the beginning of the first lesson on the topic and a discussion ensures. That same essential question is revisited during the instruction and/or post-instruction to assess student growth and learning.

- Reading and writing are an essential part of Advanced Algebra with Financial Applications. Teachers will use **written and oral response to reading** (from the textbook, newspapers, magazines, Internet, brochures, laws, etc.) as a way of assessing understanding. Some writing activities will offer students an opportunity to interpret data that is displayed in a pictorial representation. Based upon the data, they are asked to write a short, newspaper-type story centered on the graph. There is one such activity for each chapter.

HOMEWORK (15%)

- **Homework Assignments** are a daily evaluation and reflection device for both student and teacher. The level of proficiency with the homework questions should allow the teacher the opportunity to adjust the lesson as needed. The homework acts as a barometer for students, so they can formulate questions, and attempt problems on their own.

SUMMATIVE ASSESSMENTS (70%)

- **Lesson-Opener Quizzes** are short, graded, cumulative assessments that can test for prerequisite skills and/or mastery of recently taught material. These assessments are averaged and count as one full-period exam grade.

- **Full-Period Exams** are graded summative assessments that test student acquisition of skills, strategies, and content.

- **Experiential Learning** activities are project-based assessment tools that are offered to students as long-term assignments. Students are asked to do research in a variety of forms and formats in order to accomplish a task that is related to the skills, strategies, and content covered in the chapter. Their projects can be submitted in print, electronic, or presentation format. Precision and accuracy will be scrutinized during their presentations as well as the ability to use mathematical tools appropriately and strategically. Each project is valued as a single full-period exam grade.