Course No: N 730
Honors N 739

Orange Unified School District

PRE-CALCULUS WITH TRIGONOMETRY

Year Course

GRADE LEVEL: 11-12

PREREQUISITES: Algebra II with *C* or better grade.

INTRODUCTION TO THE SUBJECT:
Pre-Calculus with Trigonometry is a year course combining aspects of trigonometry, analytical geometry, functional analysis, and algebraic techniques needed in preparation for the study of calculus. The graphing calculator is used extensively in concept development as well as assignments and tests.

COURSE OBJECTIVES:

BY THE END OF THE COURSE THE STUDENT WILL BE ABLE TO:
(California Content Standards denoted. *=Trigonometry Standards, otherwise Mathematical Analysis Standards. **=Linear Algebra. ***=Probability and Statistics)

Understand the role of proof in any mathematical system, including mathematical induction. (Standard 3.0)

Graph and solve systems of linear equations and inequalities. (Standards 6.0**, 8.0**)

Analyze and graph polynomial, radical, rational, exponential, and logarithmic functions. (Standards 4.0, 6.0)

Solve applied problems utilizing the six trigonometric functions. (Standards 7.0*, 9.0*, 12.0*, 13.0*, 14.0*, 19.0*)

Compute values and graph the trigonometric functions and their inverses. (Standards 4.0*, 5.0*, 6.0*, 8.0*, 9.0*)

Understand and apply both the circular function and right triangle approaches to trigonometry. (Standards 1.0*, 2.0*, 10.0*, 19.0*)

Verify trigonometric identities. (Standards 3.0*, 10.0 *, 11.0*)

Understand and apply the concept of the limit as it applies to discreet and continuous functions. (Standard 8.0)

Compare and contrast the algebras of real numbers, complex numbers, vectors, and matrices. (Standards 1.0, 2.0, 4.0**, 5.0**, 7.0**, 8.0**, 9.0**, 10.0**, 11.0**, 12.0**, 16.0*, 17.0*, 18.0*)

Work with both the polar and rectangular forms. (Standards 1.0, 15.0*, 16.0*)

Use standard and general forms for analytical interpretation of the conic sections. (Standard 5.0)

Obtain probabilities from data. (Standards 1.0***, 2.0***, 3.0***)

Find limits and discontinuities of various algebraic expressions. (Standards 6.0, 8.0)

Compute derivatives and integrals of polynomials.

Use a graphing calculator to help solve problems, experiment, interpret results, and verify conclusions.

COURSE OVERVIEW, SCHEDULE, AND APPROXIMATE UNIT TIME ALLOTMENTS:

FIRST SEMESTER		**REGULAR DAYS**		**HONORS DAYS**	
		Block / Standard		**Block / Standard**	
I.	Techniques of Graphing Equations and Inequalities	4	8	4	8
	A. Rectangular coordinates and graphing utilities				
	B. Introduction to graphing equations				
	C. Symmetry; graphing key equations; circles				
	D. Solving equations				
	E. Solving inequalities				
	F. Lines				
II.	Analyzing Graphs of Functions	8	17	5	12
	A. Functions				
	B. Linear functions and models				
	C. Properties of functions				
	D. Library of functions; piecewise - defined functions				
	E. Graphing techniques - transformations				
	F. Operations on functions; composite functions				
	G. Mathematical models - constructing functions				

		REGULAR DAYS Block	REGULAR DAYS Standard	HONORS DAYS Block	HONORS DAYS Standard
III.	Solve and Analyzing Polynomial and Rational Functions	10	21	9	18
IV.	Exponential and Logarithmic Functions and Their Applications	7	14	6	12
V.	Circular Trigonometric Functions and Their Graphs	7	15	7	14
VI.	Inverse Trigonometric Functions and Introduction to Identities	4	10	9	20

III. Solve and Analyzing Polynomial and Rational Functions

A. Quadratic functions and models
B. Power functions and models
C. Polynomial functions and models
D. The real zeros of a polynomial function
E. Complex zeros; Fundamental Theorem of Algebra
F. Rational functions I (with limit notation used in 13.1)
G. Rational functions II - analyzing graphs
H. Types of discontinuities (see 13.3)
I. Polynomial and rational inequalities

(Recommendation: This chapter should be taught in two parts.
Part I: 3.1-3.3, 3.7, 3.8
Part II: 3.4, 3.5, 13.1, 13.3)

IV. Exponential and Logarithmic Functions and Their Applications

A. One-to-one functions; inverse functions
B. Exponential functions
C. Logarithmic functions
D. Properties of logarithm
E. Logarithmic and exponential equations
F. Compound interest
G. Growth and decay
H. Exponential, logarithmic and logistic models (optional)

V. Circular Trigonometric Functions and Their Graphs

A. Angles and their measures
B. Trigonometric functions - unit circle approach
C. Properties of the trigonometric functions
D. Graphs of the sine and cosine functions - amplitude, period, and phase shift
E. Graphs of the tangent, cotangent, cosecant, and secant function

VI. Inverse Trigonometric Functions and Introduction to Identities

A. The inverse sine, cosine and tangent
B. The inverse trigonometric functions [continued]
C. Trigonometric identities

		REGULAR DAYS		HONORS DAYS	
		Block	**Standard**	**Block**	**Standard**
D.	Sum and difference formulas				
E.	Double-angle and half-angle formulas (Honors only)				
F.	Product-to-sum and sum-to-product formulas (Honors only)				
G.	Trigonometric Equations (I and II) (Honors only)				
		40	85	40	84
Review, Field Trips, Final Exam and State Testing:		5	5	5	6
	Total:	45	90	45	90

SECOND SEMESTER

I.	Verifying Trigonometric Identities and Solving Trigonometric Equations	6	10	-	-
A.	Double-angle and half-angle formulas (Non-Honors only)				
B.	Product-to-sum and sum-to-product formulas (Non-Honors only)				
C.	Trigonometric Equations (I and II) (Non-Honors only)				
II.	Applications of Trigonometric Functions	6	12	6	12
A.	Right triangle trigonometry				
B.	The Law of Sines				
C.	The Law of Cosines				
D.	Area of a triangle				
III.	Polar Coordinates; Vectors	9	18	10	20
A.	Polar coordinates				
B.	Polar equations and graphs				
C.	The complex plane; DeMoivre's Theorem				
D.	Vectors				
E.	The dot product (optional for Non-Honors)				
F.	Vectors in space (recommended for Honors only)				
G.	The Cross Product (recommended for Honors only)				
IV.	Conic Sections	7	14	7	14
A.	Conics				
B.	The parabola				
C.	The ellipse				
D.	The hyperbola				
E.	Rotation of axes and general form of a conic				
F.	Plane curves and parametric equations				

		REGULAR DAYS Block / Standard		HONORS DAYS Block / Standard	
V.	Systems of Equations and Inequalities	2	4	2	4
	A. Solving systems of linear equations using row reduction method				
	B. Solving systems of linear equations using inverse method (2-by-2 only)				
VI.	Sequences; Induction; The Binomial Theorem	7	14	7	14
	A. Sequences				
	B. Arithmetic series				
	C. Geometric sequences; geometric series				
	D. Mathematical induction				
	E. The Binomial Theorem				
VII.	A Preview of Calculus	3	7	3	8
	A. Algebra techniques for finding limits				
	B. The tangent problem; the derivative				
	C. The area problem; the integral				
VIII.	Counting and Probability	-	-	5	10
	A. Sets and counting				
	B. Permutations and combinations				
	C. Probability of equally likely outcomes				
	D. Obtaining probabilities from data				
		40	79	40	82
Review, Field Trips, Final Exam and State Testing:		5	11	5	8
	Total:	45	90	45	90

The Orange Unified Pre-Calculus with Trigonometry course includes the California State Content Standards for Trigonometry, Linear Algebra, Probability and Statistics, and Mathematics Analysis.

DATE OF CONTENT REVISION: March 2003

DATE OF BOARD APPROVAL: May 8, 2003

Addendum
THE CALIFORNIA CONTENT STANDARDS

TRIGONOMETRY

Trigonometry uses the techniques that students have previously learned from the study of algebra and geometry. The trigonometric functions studied are defined geometrically rather than in terms of algebraic equations. Facility with these functions as well as the ability to prove basic identities regarding them is especially important for students intending to study calculus, more advanced mathematics, physics and other sciences, and engineering in college.

1.0 Students understand the notion of angle and how to measure it, in both degrees and radians. They can convert between degrees and radians.

2.0 Students know the definition of sine and cosine as *y*-and *x*-coordinates of points on the unit circle and are familiar with the graphs of the sine and cosine functions.

3.0 Students know the identity $\cos^2(x) + \sin^2(x) = 1$:

- 3.1 Students prove that this identity is equivalent to the Pythagorean theorem (i.e., students can prove this identity by using the Pythagorean theorem and, conversely, they can prove the Pythagorean theorem as a consequence of this identity).
- 3.2 Students prove other trigonometric identities and simplify others by using the identity $\cos^2(x) + \sin^2(x) = 1$. For example, students use this identity to prove that $\sec^2(x) = \tan^2(x) + 1$.

4.0 Students graph functions of the form $f(t) = A \sin(Bt + C)$ or $f(t) = A \cos(Bt + C)$ and interpret *A, B,* and *C* in terms of amplitude, frequency, period, and phase shift.

5.0 Students know the definitions of the tangent and cotangent functions and can graph them.

6.0 Students know the definitions of the secant and cosecant functions and can graph them.

7.0 Students know that the tangent of the angle that a line makes with the *x*-axis is equal to the slope of the line.

8.0 Students know the definitions of the inverse trigonometric functions and can graph the functions.

9.0 Students compute, by hand, the values of the trigonometric functions and the inverse trigonometric functions at various standard points.

10.0 Students demonstrate an understanding of the addition formulas for sines and cosines and their proofs and can use those formulas to prove and/or simplify other trigonometric identities.

11.0 Students demonstrate an understanding of half-angle and double-angle formulas for sines and cosines and can use those formulas to prove and/or simplify other trigonometric identities.

12.0 Students use trigonometry to determine unknown sides or angles in right triangles.

13.0 Students know the law of sines and the law of cosines and apply those laws to solve problems.

14.0 Students determine the area of a triangle, given one angle and the two adjacent sides.

15.0 Students are familiar with polar coordinates. In particular, they can determine polar coordinates of a point given in rectangular coordinates and vice versa.

16.0 Students represent equations given in rectangular coordinates in terms of polar coordinates.

17.0 Students are familiar with complex numbers. They can represent a complex number in polar form and know how to multiply complex numbers in their polar form.

18.0 Students know DeMoivre's theorem and can give *n*th roots of a complex number given in polar form.

19.0 Students are adept at using trigonometry in a variety of applications and word problems.

MATHEMATICAL ANALYSIS

This discipline combines many of the trigonometric, geometric, and algebraic techniques needed to prepare students for the study of calculus and strengthens their conceptual understanding of problems and mathematical reasoning in solving problems. These standards take a functional point of view toward those topics. The most significant new concept is that of limits. Mathematical analysis is often combined with a course in trigonometry or perhaps with one in linear algebra to make a yearlong pre-calculus course.

1.0 Students are familiar with, and can apply, polar coordinates and vectors in the plane. In particular, they can translate between polar and rectangular coordinates and can interpret polar coordinates and vectors graphically.

2.0 Students are adept at the arithmetic of complex numbers. They can use the trigonometric form of complex numbers and understand that a function of a complex variable can be viewed as a function of two real variables. They know the proof of DeMoivre's theorem.

3.0 Students can give proofs of various formulas by using the technique of mathematical induction.

4.0 Students know the statement of, and can apply, the fundamental theorem of algebra.

5.0 Students are familiar with conic sections, both analytically and geometrically:

5.1 Students can take a quadratic equation in two variables; put it in standard form by completing the square and using rotations and translations, if necessary; determine what type of conic section the equation represents; and determine its geometric components (foci, asymptotes, and so forth).

5.2 Students can take a geometric description of a conic section - for example, the locus of points whose sum of its distances from (1, 0) and (-1, 0) is 6 - and derive a quadratic equation representing it.

6.0 Students find the roots and poles of a rational function and can graph the function and locate its asymptotes.

7.0 Students demonstrate an understanding of functions and equations defined parametrically and can graph them.

8.0 Students are familiar with the notion of the limit of a sequence and the limit of a function as the independent variable approaches a number or infinity. They determine whether certain sequences converge or diverge.

LINEAR ALGEBRA

The general goal in this discipline is for students to learn the techniques of matrix manipulation so that they can solve systems of linear equations in any number of variables. Linear algebra is most often combined with another subject, such as trigonometry, mathematical analysis, or pre-calculus.

1.0 Students solve linear equations in any number of variables by using Gauss-Jordan elimination.

2.0 Students interpret linear systems as coefficient matrices and the Gauss-Jordan method as row operations on the coefficient matrix.

3.0 Students reduce rectangular matrices to row echelon form.

4.0 Students perform addition on matrices and vectors.

5.0 Students perform matrix multiplication and multiply vectors by matrices and by scalars.

6.0 Students demonstrate an understanding that linear systems are inconsistent (have no solutions), have exactly one solution, or have infinitely many solutions.

7.0 Students demonstrate an understanding of the geometric interpretation of vectors and vector addition (by means of parallelograms) in the plane and in three-dimensional space.

8.0 Students interpret geometrically the solution sets of systems of equations. For example, the solution set of a single linear equation in two variables is interpreted as a line in the plane, and the solution set of a two-by-two system is interpreted as the intersection of a pair of lines in the plane.

9.0 Students demonstrate an understanding of the notion of the inverse to a square matrix and apply that concept to solve systems of linear equations.

10.0 Students compute the determinants of 2 x 2 and 3 x 3 matrices and are familiar with their geometric interpretations as the area and volume of the parallelepipeds spanned by the images under the matrices of the standard basis vectors in two-dimensional and three-dimensional spaces.

11.0 Students know that a square matrix is invertible if, and only if, its determinant is nonzero. They can compute the inverse to 2 x 2 and 3 x 3 matrices using row reduction methods or Cramer's rule.

12.0 Students compute the scalar (dot) product of two vectors in *n*-dimensional space and know that perpendicular vectors have zero dot product.

PROBABILITY AND STATISTICS

This discipline is an introduction to the study of probability, interpretation of data, and fundamental statistical problem solving. Mastery of this academic content will provide students with a solid foundation in probability and facility in processing statistical information.

1.0 Students know the definition of the notion of *independent events* and can use the rules for addition, multiplication, and complementation to solve for probabilities of particular events in finite sample spaces.

2.0 Students know the definition of *conditional probability* and use it to solve for probabilities in finite sample spaces.

3.0 Students demonstrate an understanding of the notion of *discrete random variables* by using them to solve for the probabilities of outcomes, such as the probability of the occurrence of five heads in 14 coin tosses.

4.0 Students are familiar with the standard distributions (normal, binomial, and exponential) and can use them to solve for events in problems in which the distribution belongs to those families.

5.0 Students determine the mean and the standard deviation of a normally distributed random variable.

6.0 Students know the definitions of the *mean, median,* and *mode* of a distribution of data and can compute each in particular situations.

7.0 Students compute the variance and the standard deviation of a distribution of data.

8.0 Students organize and describe distributions of data by using a number of different methods, including frequency tables, histograms, standard line and bar graphs, stem-and-leaf displays, scatterplots, and box-and-whisker plots.